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Anomalous Flavor $\mathcal{U}(1)$: Predictive Texture For Bi-maximal Neutrino Mixing

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Abstract

We present a scenario which naturally provides bi-maximal neutrino mixings for a simultaneous explanation of the recent atmospheric and solar neutrino data. A crucial role is played by an anomalous flavor $\mathcal{U}(1)$ symmetry, which is also important for a natural understanding of charged fermion mass hierarchies and magnitudes of the CKM matrix elements. Within MSSM the solar neutrino problem can be resolved either through vacuum oscillations or the large mixing angle MSW solution. In supersymmetric GUTs such as $SU(5)$ and $SO(10)$ the MSW solution is realized. If the flavor $\mathcal{U}(1)$ also mediates supersymmetry breaking, the vacuum solution in MSSM is eliminated, and only the large mixing angle MSW solution survives.

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Within the various atmospheric and solar neutrino oscillation solutions allowed by the SuperKamiokande (SK) data (see [1] and [2] respectively), the bi-maximal mixing scenario is particularly attractive and much studied [3]-[6]. Recently an interesting and relatively simple texture for the neutrino mass matrix

$$\hat{M}_\nu = 3D \left(\begin{array}{ccc} 0 & m_1 & m_2 \\ m_1 & 0 & 0 \\ m_2 & 0 & 0 \end{array} \right) \tag{1}$$

has been proposed [5, 6], which naturally yields large $\nu_\mu - \nu_\tau$ mixing (if mass scales m_1, m_2 are of the same order), and maximal $\nu_e - \nu_{\mu,\tau}$ oscillations. The $\nu_\mu - \nu_\tau$ mixing can become maximal for $m_1 \simeq m_2$. In [5] an analysis of the texture in (1) was presented, but not how it was generated. In [6] a specific model realizing (1) within the standard model (MS) framework was given,=20 and requires the introduction of additional scalar doublets as well as triplet. Suitable discrete and continuous symmetries also must be imposed. It seems natural to search for an alternative, possibly a more economical framework for implementing (1).

In this paper we present a simple way for realizing (1) in which an anomalous $\mathcal{U}(1)$ flavor symmetry plays a crucial role. Atmospheric anomaly is due to large $\nu_\mu - \nu_\tau$ oscillations. Although, (1) gives maximal $\nu_e - \nu_{\mu,\tau}$ mixing to explain the solar neutrino deficit, a priori it is not clear whether this corresponds to the vacuum or large angle MSW oscillations. As we show below, this depends on the specifics of the scenario (since deviations from the zero entries in (1), determining the splitting Δm^2_{12}, are model dependent). The $\mathcal{U}(1)$ flavor symmetry also helps provide a natural understanding of the hierarchies between the charged fermion masses and the CKM matrix elements.

Before presenting the mechanism, let us demonstrate how the texture in (1) leads to bi-maximal mixings. Using an orthogonal transformation

$$U_1^T \hat{M}_\nu U_1 = 3D \hat{M'}_\nu \ , \tag{2}$$

where

$$U_1 = 3D \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_\theta & -\, s_\theta \\ 0 & s_\theta & c_\theta \end{array} \right) \ , \tag{3}$$

$$s_\theta \equiv \sin\theta \ , \quad c_\theta \equiv \cos\theta \ , \quad \tan\theta = 3D \frac{m_2}{m_1} \ , \tag{4}$$

the neutrino mass matrix takes the degenerate form

$$\hat{M'}_\nu = 3D \left(\begin{array}{ccc} 0 & m & 0 \\ m & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) , \qquad m = 3D\sqrt{m_1^2 + m_2^2} \ . \tag{5}$$

(5) is diagonalized through a transformation with maximal rotation angles

$$U_2^T \hat{M'}_\nu U_2 \equiv \hat{M}_\nu^{\rm diag} = 3D{\rm Diag}(m \ , \ \ - m \ , \ 0) \ , \tag{6}$$

where

$$U_2 = 3D \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & - \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{array} \right) \ . \tag{7}$$

Taking into account (3), (7) the neutrino mixing matrix is

$$U_\nu = 3D U_1 U_2 = 3D \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & - \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} c_\theta & \frac{1}{\sqrt{2}} c_\theta & - s_\theta \\ \frac{1}{\sqrt{2}} s_\theta & \frac{1}{\sqrt{2}} s_\theta & c_\theta \end{array} \right) \ . \tag{8}$$

From=20(8) [taking into account (4)] the atmospheric and solar neutrino oscillation amplitudes respectively are

$$\mathcal{A}(\nu_\mu \to \nu_\tau) = 3D\frac{4m_1^2 m_2^2}{(m_1^2 + m_2^2)^2} \ ,$$
$$\mathcal{A}(\nu_e \to \nu_{\mu,\tau}) = 3D1 \ , \tag{9}$$

where the oscillation amplitudes are defined as

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = 3D4 \Sigma_{j<i} U_\nu^{\alpha j} U_\nu^{\alpha i} U_\nu^{\beta j} U_\nu^{\beta i} \ , \tag{10}$$

(α, β denote flavor indices and i, j the mass eigenstates).

From=20(9) we see that for $m_1 \simeq m_2$, we have the bi-maximal oscillations scenario. As far as the neutrino mass^2 splittings are concerned, since the mass spectrum is

$$m_{\nu_1} = 3Dm_{\nu_2} = 3Dm \ , \qquad m_{\nu_3} = 3D0 \ , \tag{11}$$

we will have

$$\Delta m_{32}^2 = 3Dm^2 \ , \quad \Delta m_{21}^2 = 3D0 \ . \tag{12}$$

Having $m^2 \sim m_{\rm atm}^2 \sim 10^{-3}$ eV^2, the atmospheric neutrino puzzle is successfully resolved. However, without a non-zero $\Delta m_{21}^2 = 3D0$, the solar neutrino oscillations will be absent.

This deviation from zero will determine whether the vacuum or MSW solution is realized. In [6] the non-zero splittings emerge from radiative corrections, and led to the vacuum oscillation solution.=20 We will discuss this issue within the framework of MSSM as well as SUSY $SU(5)$. We also indicate the implications if the anomalous $\mathcal{U}(1)$ mediates in addition SUSY breaking.

1 Model with $\mathcal{U}(1)$ flavor symmetry

We introduce an anomalous $\mathcal{U}(1)$ flavor symmetry which, may arise in effective field theories from strings. The cancellation of its anomalies occurs through the Green-Schwarz mechanism [7]. Due to the anomaly, the Fayet-Iliopoulos term

$$\xi \int d^4\theta V_A \tag{13}$$

is always generated [8], where, in string theory, ξ is given by [9]

$$\xi = 3D \frac{g_A^2 M_P^2}{192\pi^2} \mathrm{Tr}Q \ . \tag{14}$$

The D_A-term will have the form

$$\frac{g_A^2}{8} D_A^2 = 3D \frac{g_A^2}{8} \left(\Sigma Q_a |\varphi_a|^2 + \xi \right)^2 \ , \tag{15}$$

where Q_a is the 'anomalous' charge of φ_a superfield.

In [10] the anomalous $\mathcal{U}(1)$ symmetry was considered as a mediator of SUSY breaking. In [11], the anomalous Abelian symmetries were exploited as flavor symmetries for a natural understanding of hierarchies of fermion masses and mixings, while in [12] the various neutrino oscillation scenarios with $\mathcal{U}(1)$ symmetry were studied.

Assuming $\mathrm{Tr}Q > 0$ ($\xi > 0$) and introducing a superfield X with $Q_X = 3D - 1$, we can ensure that the cancellation of (15) fixes the VEV of the scalar component of X to be

$$\langle X \rangle = 3D\sqrt{\xi} \ . \tag{16}$$

Further, we will assume that

$$\frac{\langle X \rangle}{M_P} \equiv \epsilon \simeq 0.2 \ . \tag{17}$$

The parameter ϵ is an important expansion parameter for understanding the magnitudes of fermion masses and mixings.

Starting our considerations with the neutrino sector within the framework of MSSM (which is more general than some specific GUT model), let us consider the following prescription for the $\mathcal{U}(1)$ charges

$$Q_X = 3D - 1 \ , \ Q_{l_2} = 3DQ_{l_3} = 3Dk \ , \ Q_{l_1} = 3Dk + n \ , \ Q_{h_u} = 3DQ_{h_d} = 3D0 \ . \tag{18}$$

In order to obtain non-zero neutrino masses, we introduce two right-handed neutrino superfields $\mathcal{N}_1, \mathcal{N}_2$, which, by a judicious choice of $\mathcal{U}(1)$ charges [3]

$$Q_{\mathcal{N}_1} = 3D - Q_{\mathcal{N}_2} = 3Dk \ , \tag{19}$$

will provide a texture similar to (1). From=20(18), (19), the relevant couplings will be:

$$\begin{array}{c c} & \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} & \begin{pmatrix} \epsilon^{2k+n} & \epsilon^n \\ \epsilon^{2k} & 0 \\ \epsilon^{2k} & 0 \end{pmatrix} \end{array} h_u \ , \qquad \begin{array}{c c} & \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{array}{c} \mathcal{N}_1 \\ \mathcal{N}_2 \end{array} & \begin{pmatrix} \epsilon^{2k} & 1 \\ 1 & 0 \end{pmatrix} \end{array} M \ , \tag{20}$$

where M is some mass scale. Integration of $\mathcal{N}_{1,2}$ states yields the neutrino mass matrix

$$\hat{M}_\nu \propto \begin{pmatrix} \epsilon^n & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m \ , \qquad m = 3D\frac{\epsilon^{2k+n} h_u^2}{M} \ , \tag{21}$$

which resembles the texture (1), but differs from it by a nonzero (1,1) entry, and provides the Δm^2_{21} splitting.

From=20(21) and (10) we have for the atmospheric and solar neutrino oscillation parameters (respectively)

$$\begin{aligned} \Delta m^2_{32} \equiv m^2_{\mathrm{atm}} = 3Dm^2 \sim 10^{-3}\ \mathrm{eV}^2 \ , \\ \mathcal{A}(\nu_\mu \to \nu_\tau) \sim 1 \ , \end{aligned} \tag{22}$$

$$\begin{aligned} \Delta m^2_{21} \simeq 2 m^2_{\mathrm{atm}} \epsilon^n \ , \\ \mathcal{A}(\nu_e \to \nu_{\mu,\tau}) = 3D1 - \mathcal{O}(\epsilon^{2n}) \ . \end{aligned} \tag{23}$$

[3]For models in which $\mathcal{U}(1)$ flavor symmetry plays a crucial role for achieving maximal/large mixings, see refs. [13, 4].

We observe that the mass^2 splitting for solar neutrinos is expressed in terms of the atmospheric scale $m_{\rm atm}$ and n-th power of ϵ.

From=20(23) we have
=20

$$\Delta m_{21}^2 \propto \left\{ \begin{array}{ll} 10^{-10}\ {\rm eV}^2 & {\rm for}\ n=3D10 \\ 10^{-3}\ {\rm eV}^2 & {\rm for}\ n=3D3 \end{array} \right. \tag{24}$$

Therefore, $n=3D10$ corresponds to vacuum oscillations of solar neutrinos, while $n=3D3$ gives the large angle MSW solution. The MSSM does not constrain n to be either 10 or 3, and so both scenarios are possible. To see this, let us consider the charged fermion sector. With the prescription

$$Q_{e_3^c}=3Dp\ ,\quad Q_{e_2^c}=3Dp+2\ ,\quad Q_{e_1^c}=3Dp+5-n\ , \tag{25}$$

the Yukawa couplings for charged leptons have the form =20

$$\begin{array}{cc} & \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} & \left(\begin{array}{ccc} \epsilon^5 & \epsilon^{n+2} & \epsilon^n \\ \epsilon^{5-n} & \epsilon^2 & 1 \\ \epsilon^{5-n} & \epsilon^2 & 1 \end{array} \right) \epsilon^{p+k} h_d\ , \end{array} \tag{26}$$

providing the desirable hierarchies

$$\lambda_\tau \sim \epsilon^{p+k}\ ,\quad \lambda_e:\lambda_\mu:\lambda_\tau \sim \epsilon^5:\epsilon^2:1\ , \tag{27}$$

with

$$\tan\beta \sim \epsilon^{p+k}\frac{m_t}{m_b}\ . \tag{28}$$

As far as the quark sector is concerned, with

$$\begin{array}{c} Q_{q_3}=3D0\ ,\quad Q_{q_2}=3D2\ ,\quad Q_{q_1}=3D3\ ,\quad Q_{d_3^c}=3DQ_{d_2^c}=3Dp+k\ , \\ Q_{d_1^c}=3Dp+k+2\ ,\quad Q_{u_3^c}=3D0\ ,\quad Q_{u_2^c}=3D1\ ,\quad Q_{u_1^c}=3D3\ , \end{array} \tag{29}$$

the appropriate Yukawa couplings are

$$\begin{array}{cc} & \begin{array}{ccc} d_1^c & d_2^c & d_3^c \end{array} \\ \begin{array}{c} q_1 \\ q_2 \\ q_3 \end{array} & \left(\begin{array}{ccc} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{array} \right) \epsilon^{p+k} h_d\ , \end{array} \tag{30}$$

$$\begin{array}{cc} & \begin{array}{ccc} u_1^c & u_2^c & u_3^c \end{array} \\ \begin{array}{c} q_1 \\ q_2 \\ q_3 \end{array} & \begin{pmatrix} \epsilon^6 & \epsilon^4 & \epsilon^3 \\ \epsilon^5 & \epsilon^3 & 2 \\ \epsilon^3 & \epsilon & 1 \end{pmatrix} h_u \ , \end{array} \tag{31}$$

yielding

$$\lambda_b \sim \epsilon^{p+k} \ , \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^2 : 1 \ , \tag{32}$$

$$\lambda_t \sim 1 \ , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^3 : 1 \ . \tag{33}$$

From=20(30), (31), for the CKM matrix elements we find

$$V_{us} \sim \epsilon \ , \quad V_{cb} \sim \epsilon^2 \ , \quad V_{ub} \sim \epsilon^3 \ . \tag{34}$$

We see that the MSSM does not fix the values of n, p, k.=20 However, specific GUTs can be more restrictive. To demonstrate this, we consider the simplest version of $SU(5)$ GUT, with three families of $(10 + \bar{5})$-plets. Due to these unified multiplets:

$$Q_q = 3DQ_{e^c} = 3DQ_{u^c} = 3DQ_{10} \ , \qquad Q_l = 3DQ_{d^c} = 3DQ_{\bar{5}} \ . \tag{35}$$

The known hierarchies (34) of the CKM matrix elements now fix the relative charges of 10-plets,

$$Q_{10_3} = 3D0 \ , \quad Q_{10_2} = 3D2 \ , \quad Q_{10_1} = 3D3 \ , \tag{36}$$

while (27), (32) dictate

$$Q_{\bar{5}_3} = 3DQ_{\bar{5}_2} = 3Dk \ , \quad Q_{\bar{5}_1} = 3Dk + 2 \ . \tag{37}$$

Comparing (35)-(37) with (18), (25) (29) we see that the minimal $SU(5)$ GUT fixes n and p to be

$$n = 3D2 \ , \qquad p = 3D0 \ , \tag{38}$$

From=20(23) (which now turns out to be predictive since n is fixed) we get

$$\Delta m_{21}^2 \sim 10^{-4} \ \mathrm{eV}^2 \ . \tag{39}$$

This value is close to the scale corresponding to the large angle MSW oscillations of the solar neutrinos. We see that our mechanism within $SU(5)$ GUT strongly suggests large angle MSW oscillations for solar neutrinos.

The same conclusion can be reached with $SO(10)$ GUT, since also in this case the prescription of the $\mathcal{U}(1)$ charges [4] would exclude the vacuum solution of the solar neutrino problem. Within this framework the large $\nu_\mu - \nu_\tau$ mixing remains unchanged.

Let us note that the conclusions presented above are valid if the anomalous $\mathcal{U}(1)$ only acts as flavor symmetry and is not tied with SUSY breaking. In several models an anomalous $\mathcal{U}(1)$ symmetry also acts as a mediator of SUSY breaking [10]. This can be very useful for adequate suppression of FCNC [14, 15] and dimension five nucleon decay [15]. In this type of scenarios the soft mass^2 for sparticles, which have non-zero $\mathcal{U}(1)$ charges, emerge from non-zero D_A-term and equal

$$m^2_{\tilde{\phi}_i} = 3Dm^2_S Q_{\phi_i} \ , \tag{40}$$

where m_S is taken $\mathcal{O}(10\ \mathrm{TeV})$. Therefore,=20 the $\mathcal{U}(1)$ charges of matter superfields must be positive in order to avoid $SU(3)_c \times U(1)_{\mathrm{em}}$ breaking. On the other hand, from (25) we see that the choice $n = 3D10$ is excluded (!) [since $0 \leq p + k \leq 3$ $(1 \gtrsim \lambda_{\tau,b} \lesssim 10^{-2})$], which means that in this case the vacuum oscillation solution for solar neutrinos is not realized even within the MSSM framework. The cases $n = 3D2, 3$, which correspond to the large angle MSW solution, are still allowed. The interesting point is that the neutrino oscillation scenario is linked with SUSY breaking mechanism.

In conclusion, we have suggested a scenario for obtaining bi-maximal neutrino mixing. A crucial role is played by an anomalous $\mathcal{U}(1)$ flavor symmetry for obtaining the simple neutrino mass matrix texture in (1). The atmospheric neutrino puzzle is resolved by large/maximal $\nu_\mu - \nu_\tau$ mixing, while the scenario for large angle $\nu_e - \nu_{\mu,\tau}$ oscillations is model dependent: within the MSSM, both the large angle vacuum or the large angle MSW oscillations are possible, while the $SU(5)$ GUT (and also $SO(10)$) predicts the large angle MSW solution. If the anomalous $\mathcal{U}(1)$ flavor symmetry is also a mediator of SUSY breaking, then the solar neutrino vacuum oscillations are excluded and the large angle MSW solution is responsible for the solar neutrino deficit. Finally, the $\mathcal{U}(1)$ flavor symmetry also nicely explains the hierarchies between the charged fermion masses and the magnitudes of the CKM matrix elements.

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